

QUEEN MARY
UNIVERSITY OF LONDON
BSc EXAMINATION 2007

MAS/118 Differential Equations

??:?? ?m, ??day ??th May, 2007

Duration: 2 hours

*You should attempt all questions. Marks awarded are shown next to the question.
Calculators are NOT permitted in this examination. The unauthorised use of a
calculator constitutes an examination offence.*

Problem 1

- a) Determine the solution of the initial value problem

$$y' = \frac{1}{x^2 - 5x + 6}, \quad y(0) = -1$$

by integration. (9 points)

- b) Determine the solution of the initial value problem

$$y' = \frac{xe^y}{x^2 + 1}, \quad y(0) = 0$$

by separation of variables. (10 points)

- c) Show that $y = \sqrt{x^3 + x}$ is a particular solution of the differential equation

$$y' = \frac{3y}{2x} - \frac{1}{y}$$

(9 points)

Problem 2

Consider the inhomogeneous linear differential equation

$$y' = -\frac{x}{1+x}y + (1+x)e^{-x}$$

- a) For initial condition $y(0) = 1$ compute the solution of the differential equation. (10 points)
- b) Determine the value of the slope of your solution at $x = 0$ and the limit of $y(x)$ when $x \rightarrow \infty$. (6 points)
- c) Sketch the solution in a diagram. (6 points)

Problem 3

- a) Compute the general solution of the linear second order differential equation

$$2y'' + 3y' + y = 0$$

(5 points)

- b) Compute a particular solution of the inhomogeneous differential equation

$$2y'' + 3y' + y = e^{-x}$$

and hence write down the general solution of this differential equation.

(8 points)

- c) Fix the constants of integration of the solution of part b) according to the initial condition $y(0) = 1$, $y'(0) = -2$ and sketch this solution in a diagram.

(8 points)

Problem 4

Consider the linear two-dimensional system of differential equations

$$y_1' = y_1 + 4y_2, \quad y_2' = -2y_1 - 5y_2$$

- a) Solve the eigenvalue problem of the corresponding two-dimensional matrix, i.e. compute the eigenvalues and the eigenvectors.

(8 points)

- b) Determine the general solution of the system of differential equations.

(2 points)

- c) Fix the constants of integration in such a way that the solution fulfils the initial condition $y_1(0) = 1$ and $y_2(0) = 0$. Write down the corresponding solution $y_1(x)$ and $y_2(x)$ of the system of differential equations.

(7 points)

- d) Compute the slope of the solution $y_1(x)$ and $y_2(x)$ of part c) at $x = 0$.

(4 points)

- e) Sketch the solution of part c) in a diagram.

(8 points)